

About Inefficient Frequency

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Inefficient frequency was first noticed in [1]. Here we have a more general model in which different frequency can have different idle power.

1 What is inefficient frequency?

A frequency f_1 is inefficient with respect to a higher frequency f_2 if for any task, running at frequency f_1 consumes more energy than running at frequency f_2 . This means that running at lower frequency results in longer execution time and more energy consumption. Therefore, we should never use inefficient frequency in the system.

2 What is the sufficient and necessary condition for inefficient frequency?

Theorem 1 Frequency f_1 is inefficient with respect to frequency f_2 ($f_1 < f_2$) iff $\frac{P(f_1)-I(f_2)}{f_1} \geq \frac{P(f_2)-I(f_2)}{f_2}$.

Proof: (simplified version) Let the computational requirement of the task be C cycles. If frequency f_1 is used to execute the task, then the execution time is $\frac{C}{f_1}$, and the energy consumption is $\frac{C \cdot P(f_1)}{f_1}$. If frequency f_2 is used, the execution time is $\frac{C}{f_2}$ which is less than $\frac{C}{f_1}$. Thus, the CPU has to idle for $\frac{C}{f_1} - \frac{C}{f_2}$ time during which it consumes idle power $I(f_2)$. Therefore the energy consumption when using frequency f_2 is $\frac{C \cdot P(f_2)}{f_2} + I(f_2) \left(\frac{C}{f_1} - \frac{C}{f_2} \right)$.

If f_1 is inefficient with respect to f_2 , then we have

$$\frac{C \cdot P(f_1)}{f_1} \geq \frac{C \cdot P(f_2)}{f_2} + I(f_2) \left(\frac{C}{f_1} - \frac{C}{f_2} \right)$$

which can be arranged as

$$\frac{P(f_1) - I(f_2)}{f_1} \geq \frac{P(f_2) - I(f_2)}{f_2}$$

□

3 The transitivity of efficient frequencies

Theorem 2 If frequency f_1 is efficient with respect to f_2 and f_2 is efficient with respect to f_3 ($f_1 < f_2 < f_3$), then f_1 is efficient with respect to f_3 .

Proof: Since f_1 is efficient with respect to f_2 , we have

$$\frac{P(f_2) - I(f_2)}{f_2} > \frac{P(f_1) - I(f_2)}{f_1} \left(\implies P(f_2) > \frac{f_2}{f_1} (P(f_1) - I(f_2)) + I(f_2) \right)$$

and because f_2 is efficient with respect to f_3 , we have

$$\frac{P(f_3) - I(f_3)}{f_3} > \frac{P(f_2) - I(f_3)}{f_2}$$

and continue from the above

$$> \frac{P(f_1) - I(f_2) + \frac{f_1}{f_2}(I(f_2) - I(f_3))}{f_1} > \frac{P(f_1) - I(f_3)}{f_1}$$

which proves the claim. □

References

- [1] A. Miyoshi, C. Lefurgy, E. V. Hensbergen, R. Rajamony, and R. Rajkumar. Critical Power Slope: Understanding the runtime effects of Frequency Scaling. In *Proceedings of the 16th ACM International Conference on Supercomputing*, June 2002.